2. MECHANICS

UNIT OUTLINE

Section	Recommended Time	Giancoli Sections
2.1 Kinematics AHL 9.1 Projectile Motion	6h 2h	2.1-2.5, 3.8 3.5, 3.6
2.2 Forces and Dynamics	6h	4.-4.6, 7.1-7.3
2.3 Work, Energy, Power	3h	6.1-6.4, 6.8, 6.10
2.4 Uniform Circular Motion	2h	5.1, 5.2
6.1 Gravitational Force & Field AHL 9.2 Gravitational Fields AHL 9.4 Orbital Motion	2h 2h 2h	5.6-5.8 5.7 5.8

2.1 KINEMATICS

TERMS

- A **scalar** quantity has magnitude
- A **vector** quantity has magnitude and direction

- A **qualitative** description uses words to describe qualities
- A **quantitative** description uses numerical values

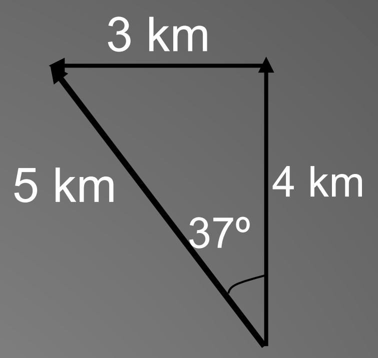
DISPLACEMENT

- Displacement is a vector quantity
- **Displacement** is the distance and direction from a known starting point
- Displacement is measured in metres

For example, if you travel

4 km North then 3 km West

your displacement (from your starting position) is 5 km at 37° West of North



VELOCITY

- Velocity is a vector quantity
- **Velocity** is the speed and the direction from a known origin
- Velocity is measured in ms^{-1}

$$v = \frac{\Delta d}{\Delta t}$$

v = velocity, $m.s^{-1}$

Δd = change in displacement, m

Δt = change in time, s

ACCELERATION

- Acceleration is a vector quantity
- **Acceleration** is the change in velocity per unit time
- Acceleration is measured in ms^{-2}
- Acceleration can be a change in direction OR speed OR both
- A deceleration is the same as a negative acceleration

$$a = \frac{\Delta v}{\Delta t}$$

a = acceleration, $m.s^{-2}$

Δv = change in velocity, $m.s^{-1}$

Δt = change in time, s

AVERAGE & INSTANTANEOUS

- The average velocity is calculated over a period of time
- **Average velocity** is total distance divided by total time

$$v_{av} = \frac{\Delta d}{\Delta t}$$

$\boldsymbol{v_{av}}$ = average velocity, $m.s^{-1}$

$\boldsymbol{\Delta d}$ = total displacement, m

Δt = time taken, s

AVERAGE & INSTANTANEOUS

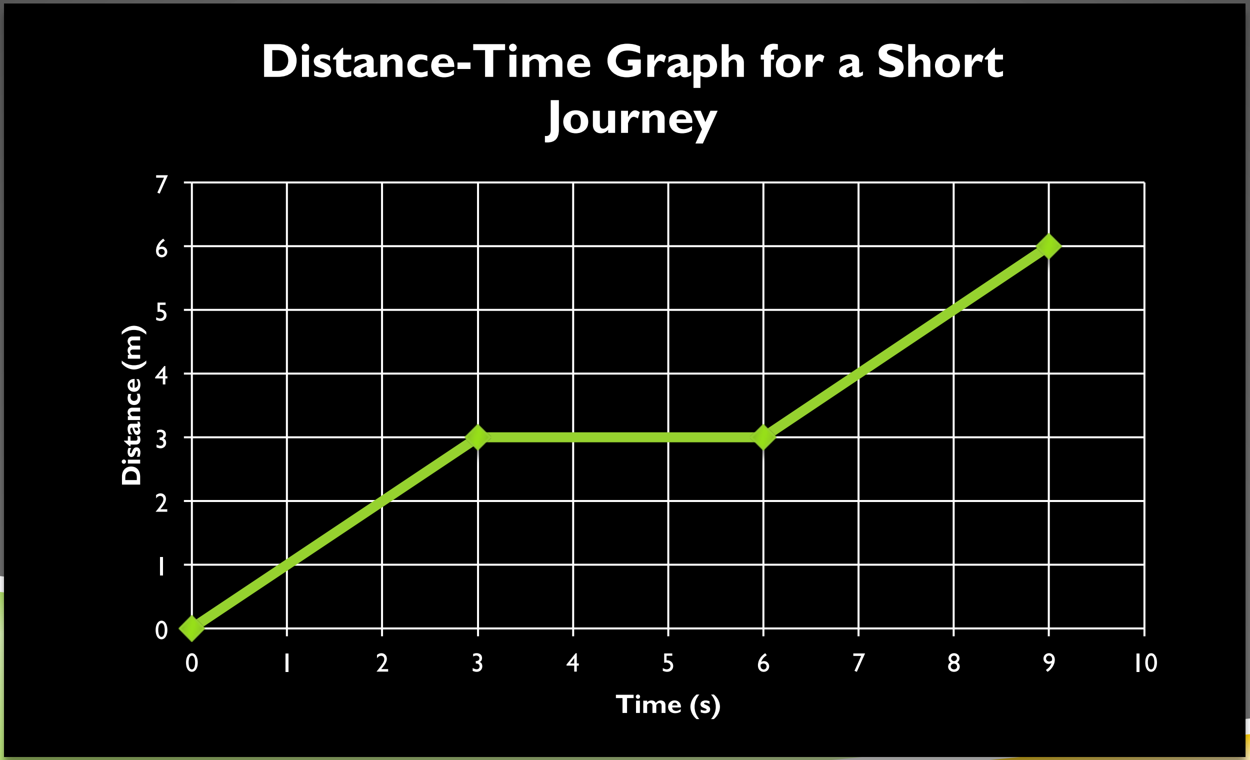
- Instantaneous velocity is the velocity at a particular point in time
- **Instantaneous velocity** is calculated by the limit as Δt approaches zero (i.e. it is the derivative of distance with time)

$$v = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

- Likewise, we can consider average and instantaneous acceleration, etc.

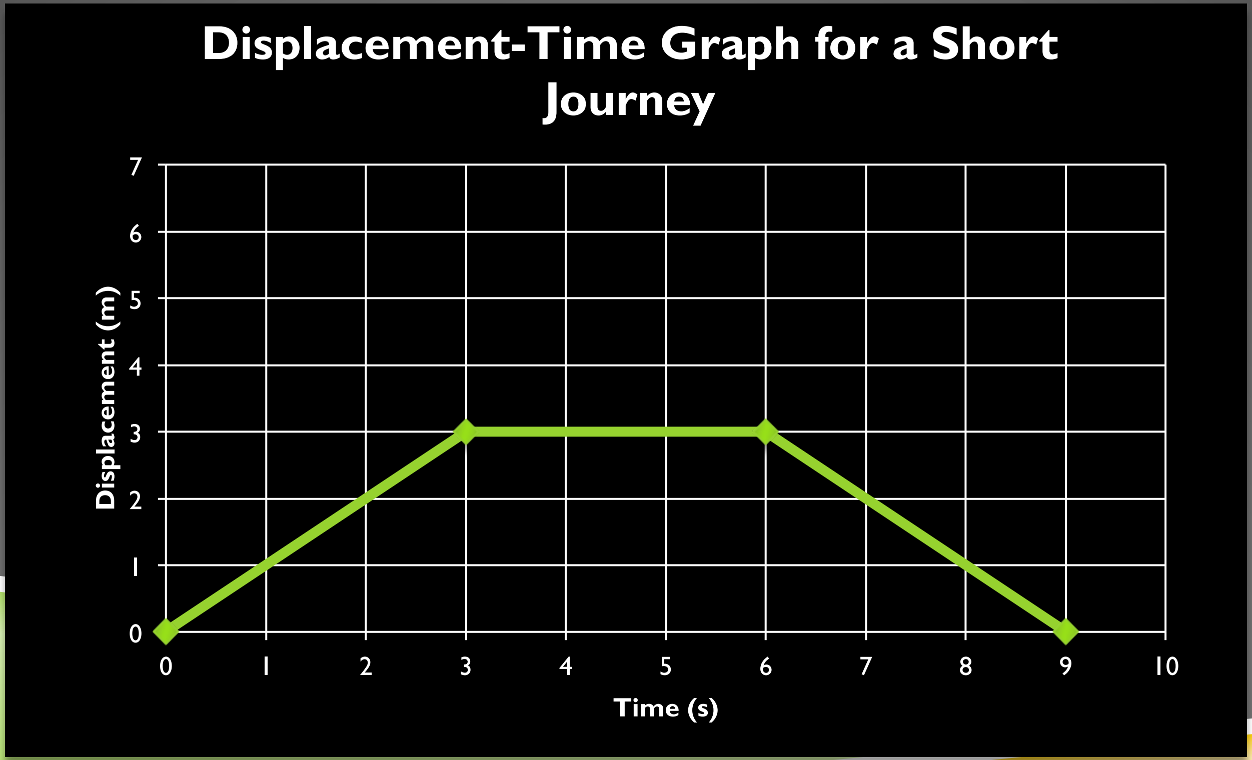
DISTANCE-TIME GRAPHS

- A **distance-time graph** has distance travelled on the y-axis and time on the x-axis
- The slope of the graph gives the **speed** of the object



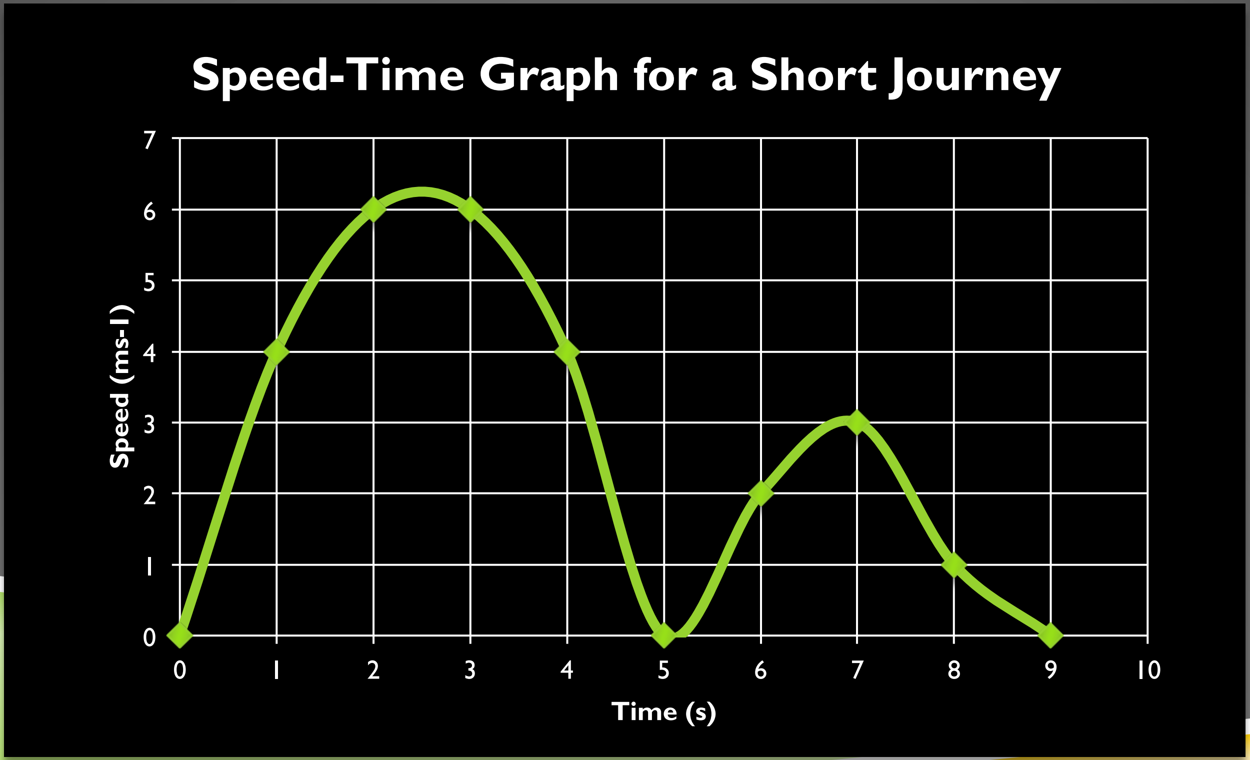
DISPLACEMENT-TIME GRAPHS

- A **displacement-time graph** has displacement on the y-axis and time on the x-axis
- The slope of the graph gives the **velocity** of the object



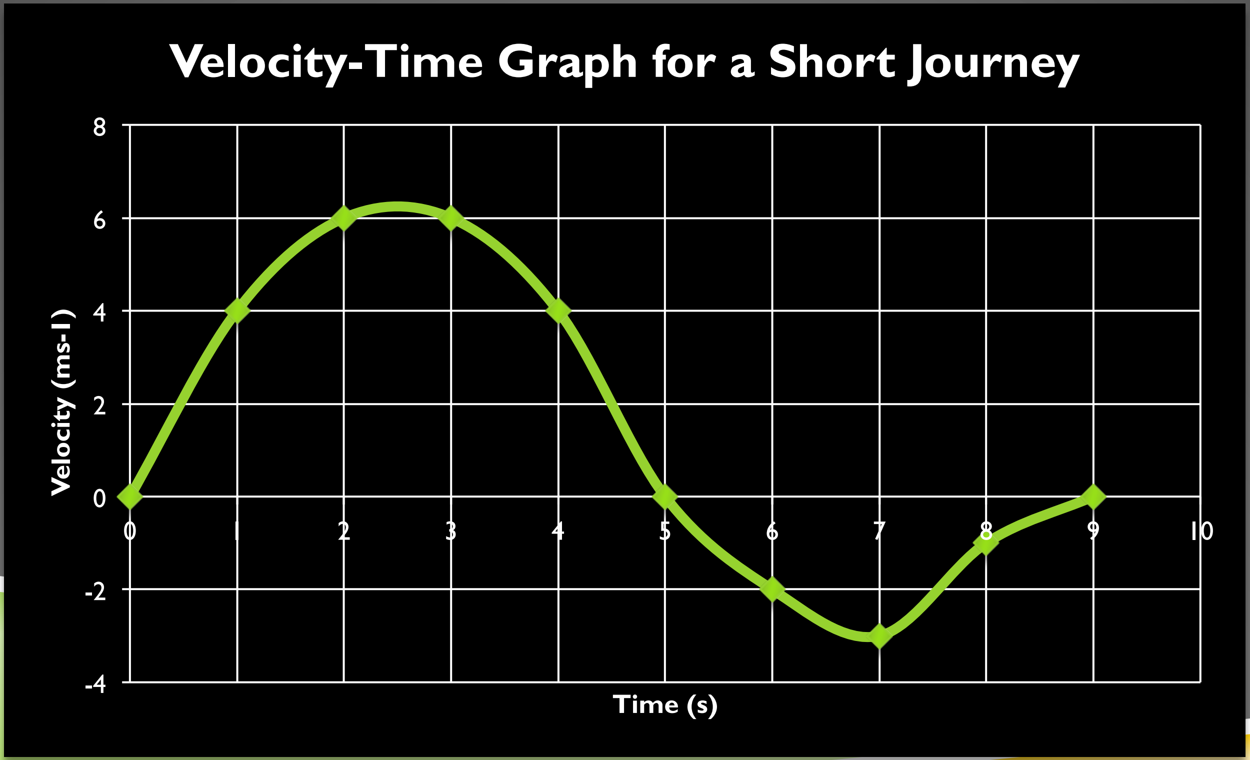
SPEED-TIME GRAPHS

- A **speed-time graph** has speed on the y-axis and time on the x-axis



VELOCITY-TIME GRAPHS

- A **veolcity-time graph** has velocity on the y-axis and time on the x-axis
- The slope of the graph gives the **acceleration**
- The area under the graph gives the **displacement**



ACCELERATION-TIME GRAPHS

- A **acceleration-time graph** has acceleration on the y-axis and time on the x-axis
- The area under the graph gives the **velocity**
- Acceleration is a vector quantity and so can be both positive and negative

GRAVITY AND AIR RESISTANCE

In the absence of air resistance, the **acceleration due to gravity** near the surface of the Earth (g) is 9.81 $m.s^{-2}$. The acceleration due to gravity decreases with distance from the centre of the Earth.

A retarding force (often called 'drag') occurs when an object moves through the air. This force depends on the shape and speed of the object. If an object falls from a great height, it will reach a speed when the retarding force is equal to the acceleration due to gravity. This is called **terminal velocity**. Terminal velocity for a person is about 60 $m.s^{-1}$.

KINEMATICS EQUATIONS

- The **kinematics equations** apply for **constant acceleration**

$$s = \frac{u+v}{2}t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$v = u + at$$

s = displacement, m

u = initial velocity, $m.s^{-1}$

v = final velocity, $m.s^{-1}$

a = acceleration, $m.s^{-2}$

t = time, s

RELATIVE MOTION

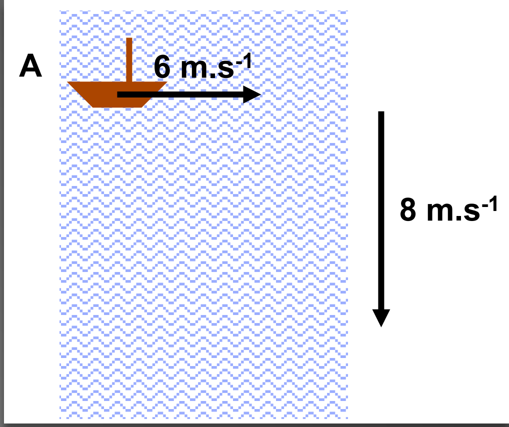
- **Relative velocity** is the velocity of one object relative to another

For example:

A boat is travelling across a river with velocity 6 ms^{-1} relative to the water

The river is flowing downstream at a velocity of 8 ms^{-1} relative to the bank

The velocity of the boat relative to the bank is calculated by vector addition



2.2 FORCES AND DYNAMICS

MASS AND WEIGHT

- **Mass** is the amount of matter in an object, measured in kg
- Mass is a measure of an object's inertia
  - **Inertial mass** is the mass that gives rise to inertia
  - **Gravitational mass** is the mass that responds to gravity
  - They are considered to be equivalent
- **Weight** is the force that gravity exerts on the mass of an object

$$F_W = mg$$

FORCE

- A **force** causes an object to change speed, direction or shape
- A force causes an acceleration in an object that is free to move
- Force is measured in Newtons, N
- Scientists consider that there are four fundamental forces:
  - **Gravity** – the weakest, acts between all particles in the universe
  - **Weak Interaction** – 10^{26} times stronger than gravity, responsible for aspects of nuclear decay
  - **Electromagnetic Force** – 10^{37} times stronger than gravity, occurs as the result of electric charge
  - **Strong Nuclear Interaction** – 10^{39} times stronger than gravity, holds protons and neutrons together in the nucleus, acts over very short distances

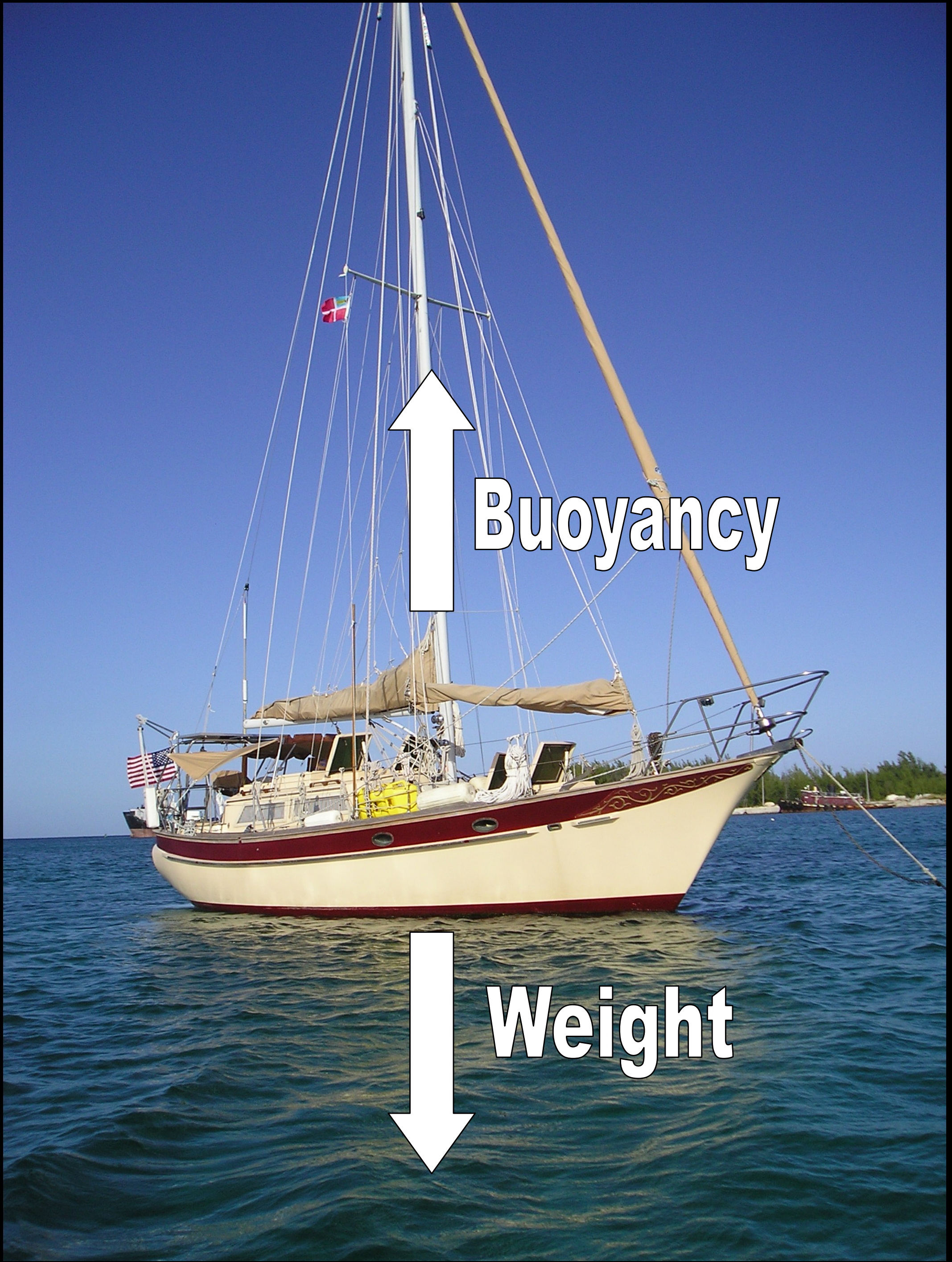
FREE-BODY FORCE DIAGRAMS

A free-body force diagram shows an object and all of the forces acting on it. The forces are represented by arrows. The direction of the arrow corresponds to the direction of the force, and the size of the arrow indicates the size of the force. The arrows must be labelled.

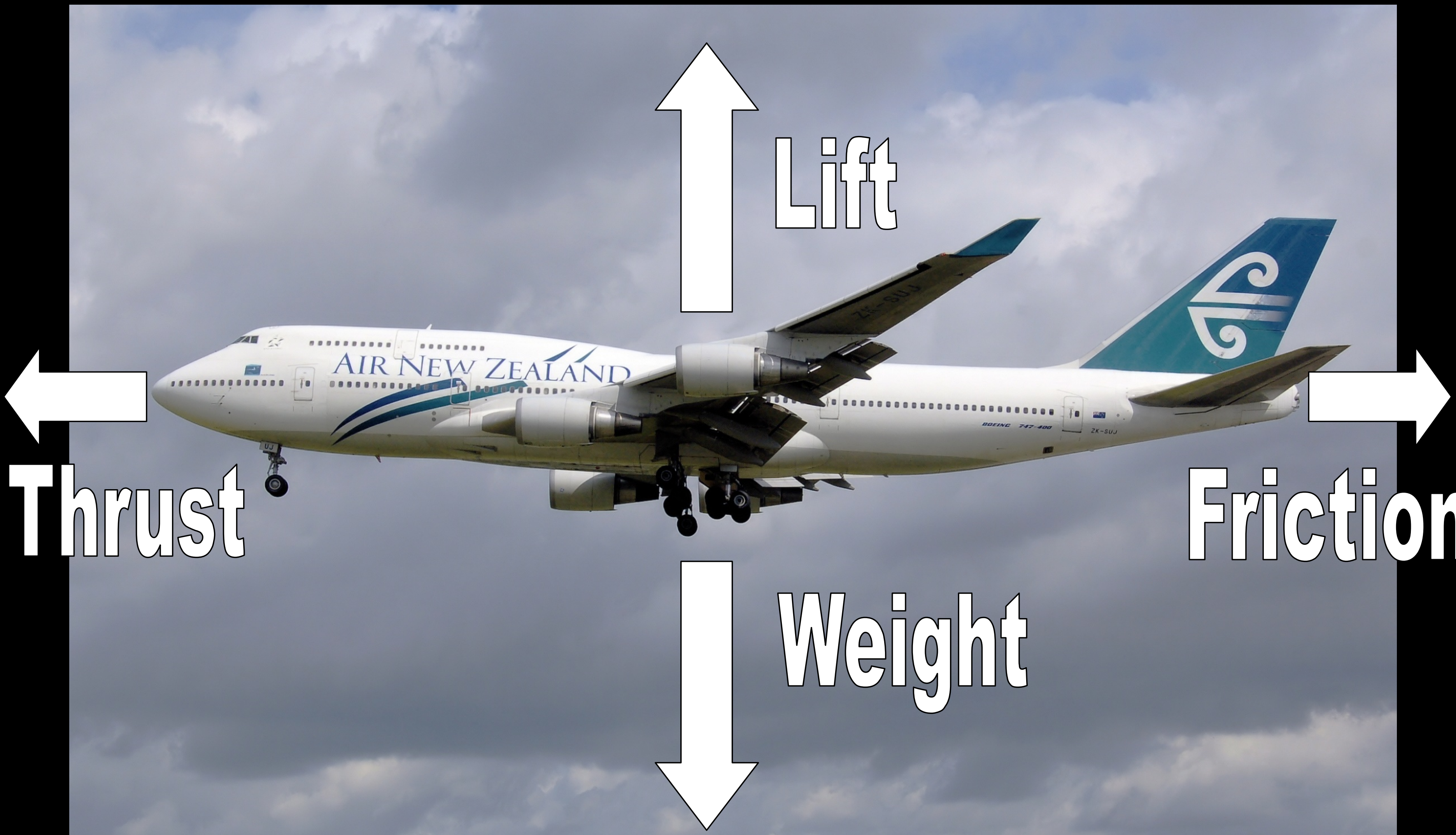
Some common forces include: weight, support, thrust, drag (or friction), lift, tension.

NEWTON'S FIRST LAW OF MOTION

- If the sum of the forces on an object is zero, that object will be stationary or travel at a constant velocity.
- An object will continue in a state of rest or uniform (linear) motion unless a net external force acts upon it


Buoyancy
Weight

Newton's First Law – Balanced Forces



TRANSLATIONAL EQUILIBRIUM

- An object is in **translational equilibrium** when the sum of the (linear) forces acting on it is zero
- If an object is stationary, it is in **static equilibrium**
- If an object is moving at constant velocity, it is in **dynamic equilibrium**

NEWTON'S SECOND LAW OF MOTION

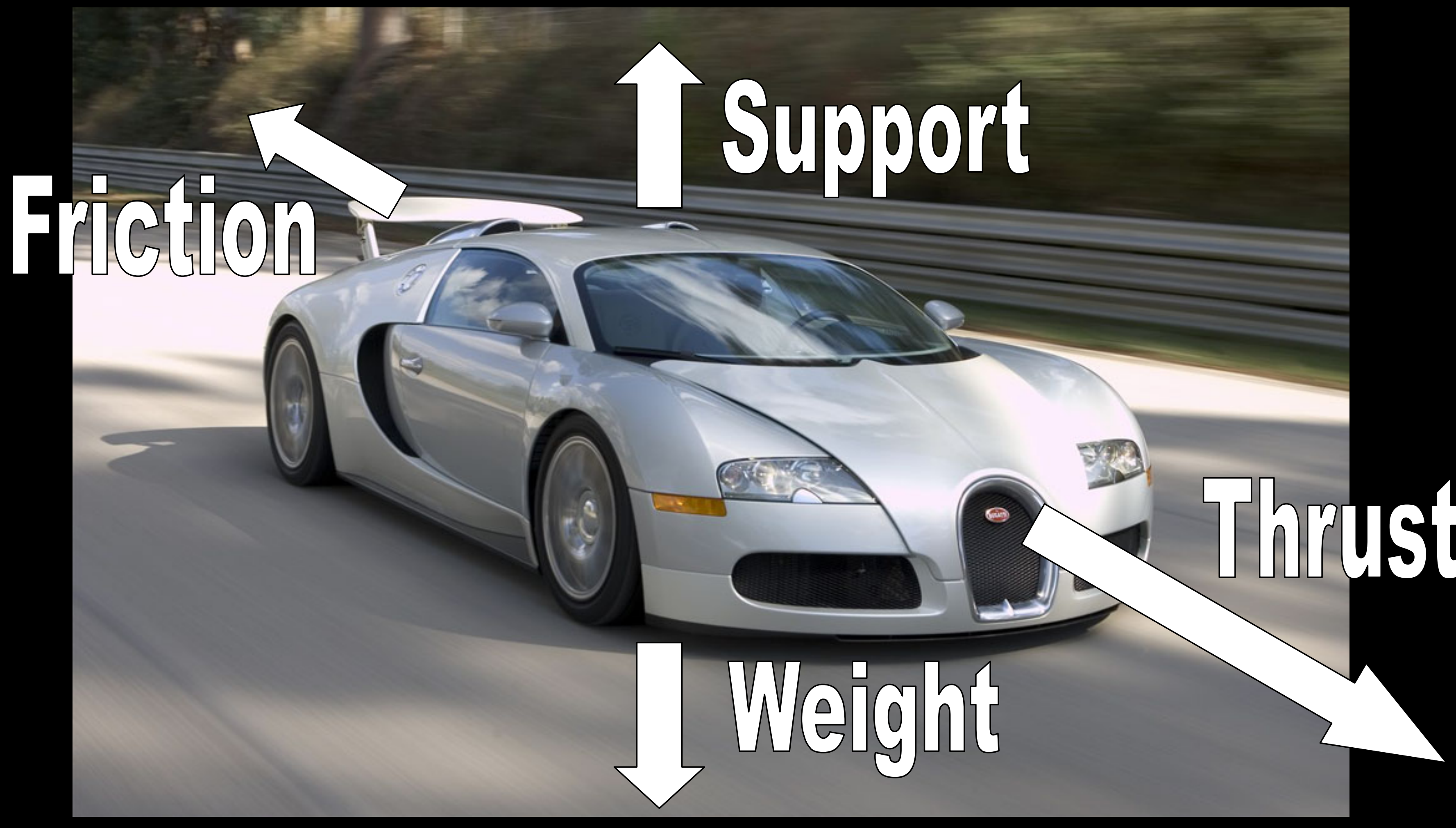
- If the sum of the forces on an object is not zero, the object will accelerate in the direction of the net force.
- The acceleration is proportional to the net force.

$$F = ma$$

F = force, N

m = mass, kg

a = acceleration caused by the force, $m.s^{-2}$


Support
Friction
Thrust
Weight

LINEAR MOMENTUM

- **Momentum** is a measure of how hard it is to start or stop an object moving
- Momentum is a vector quantity, measured in Ns or $kgms^{-1}$

$$p = mv$$

p = momentum, $kg.m.s^{-1}$ or N.s

m = mass, kg

v = velocity, $m.s^{-1}$

NEWTON'S 2ND LAW RE-WRITTEN

If we substitute $a = \Delta v/\Delta t$ into $F = ma$ we get $F = m\Delta v/\Delta t$

Since $p = mv$ we get $F = \Delta p/\Delta t$

This can be written $\Delta p = F\Delta t$, which is the equation for impulse

IMPULSE

- **Impulse** is the change in momentum

$$\Delta p = F\Delta t$$

- Impulse is the area under a force-time graph

I = impulse, $kg.m.s^{-1}$

Δp = change in momentum, $kg.m.s^{-1}$

F = force, N

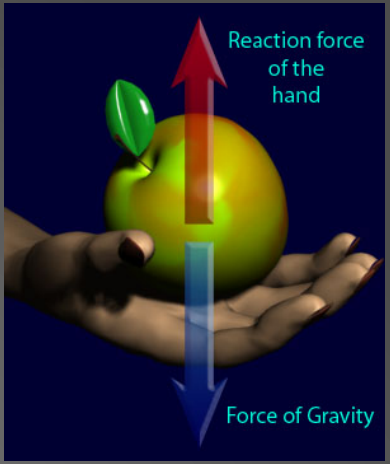
Δt = time that the force acts over, s

CONSERVATION OF MOMENTUM

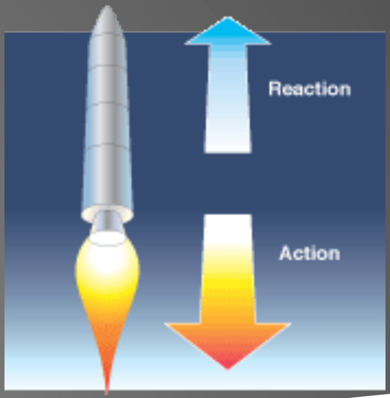
- An **isolated system** is a group of interacting objects that are not affected by any forces outside the system
- Momentum is conserved in collisions (and explosions) in isolated systems, i.e. the total momentum before a collision is equal to the total momentum after the collision
- So, if the total external force acting on a system is zero, the momentum of the system is unchanged
- When studying conservation of momentum in one-dimension, we have to assign a sign (+ or -) to the momentum of the objects
- When studying conservation of momentum in two-dimensions, we use vector addition (or vector subtraction)

NEWTON'S THIRD LAW OF MOTION

- For every action there is an equal and opposite reaction



For example, when your weight pushes into the floor (action), the structure of the floor provides a support force (reaction) that is the same magnitude ('equal') but in the opposite direction ('opposite')



For example, when rockets of a spaceship push down on the ground (action), the spaceship is propelled upwards (reaction)

2.3 WORK, ENERGY & POWER

ENERGY

- **Energy** is the ability to do work
- Energy is measured in Joules
- There are many different types of energy, including kinetic energy, radiant energy (heat and light), sound energy, thermal energy, gravitational potential energy, chemical potential energy, electrical potential energy, nuclear potential energy, elastic potential energy.

TRANSFERS AND TRANSFORMS

- An **energy transfer** occurs when energy is transferred from one object to another, but the form of the energy stays the same
- For example, a moving foot (kinetic energy) hits a stationary ball, causing the ball to move (kinetic energy)
- An **energy transform** occurs when energy is converted from one form to another
- For example, a ball is kicked up into the air and the kinetic energy of the ball is converted to gravitational potential energy

CONSERVATION OF ENERGY

- In an isolated system, energy is conserved. This means that the total energy of a system before an interaction is the same as the total energy of the system after the interaction.
- For example, when a ball is dropped, gravitational potential energy is converted to kinetic energy as the ball falls, kinetic energy is converted to elastic potential energy (and heat and sound) as the ball is compressed, elastic potential energy is converted to kinetic energy (and heat) as the ball bounces back up

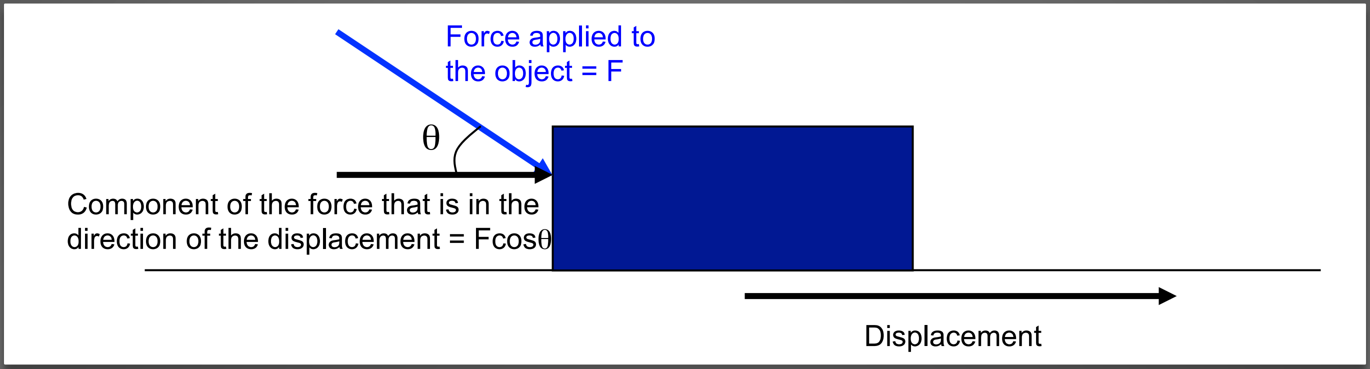
WORK

- **Work** is the transfer of energy from one form to another
- Work is measured in Joules

$$W = Fd$$

- If the force is applied at an angle, the equation becomes

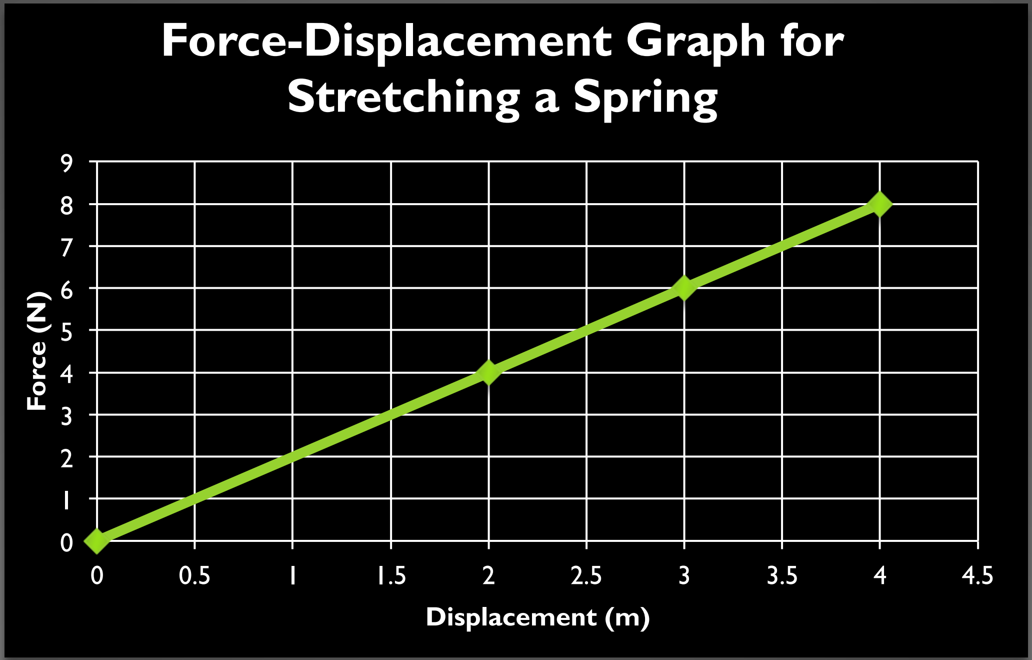
$$W = Fd\cos\theta$$



W = work, J F = force, N d = displacement, m θ = angle between the F and d

FORCE-DISPLACEMENT GRAPHS

- Work done is the area under a force-displacement graph



KINETIC ENERGY

- **Kinetic Energy** is the energy of moving objects

$$E_K = \frac{1}{2}mv^2$$

E_K = kinetic energy, J

m = mass, kg

v = velocity, $m.s^{-1}$

- Worksheet: Kinetic Energy

GRAVITATIONAL POTENTIAL ENERGY

- **Gravitational potential energy** is the energy an object has when it is raised to a height

$$E_p = mgh$$

E_P = gravitational potential energy, J

m = mass, kg

g = acceleration due to gravity, $m.s^{-2}$ (9.81 $m.s^{-2}$ at the surface of the Earth)

h = height of the object, m

- Worksheet: Gravitational potential energy

HOOKE'S LAW

- Hooke discovered a linear relationship between the extension of a spring and the restoring force of the spring

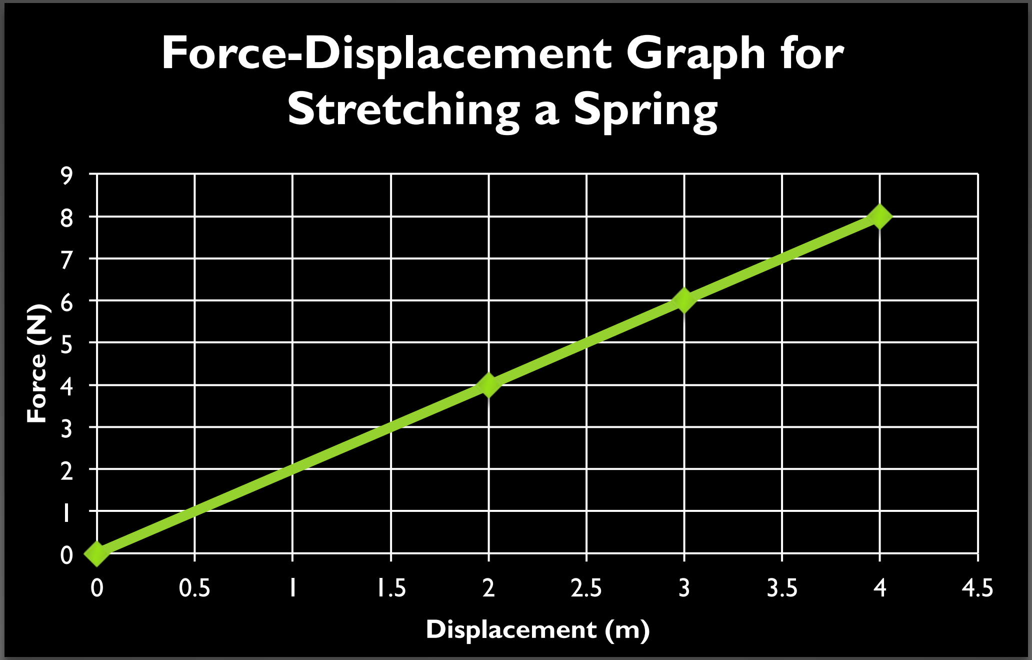
$$F = -kx$$

F = restoring force of the spring, N

k = spring constant, $N.m^{-1}$

x = displacement of the spring, m

HOOKE'S LAW



- The spring constant is given by the gradient
- Work done is given by the area under the graph

ELASTIC POTENTIAL ENERGY

- When a spring is stretched, energy is stored in the spring
- If the spring is released, this energy is converted to kinetic energy

$$E_P = \frac{1}{2}kx^2$$

- This can also be seen from the force-displacement graph for a spring

E_P = elastic potential energy, J

k = spring constant, $N.m^{-1}$

x = displacement of the spring, m

- Worksheet: Elastic Potential Energy

ELASTIC AND INELASTIC

- An **elastic** collision is one in which mechanical energy is conserved
- An **inelastic** collision is one in which some of the mechanical energy is converted to other forms, e.g. heat, so the final kinetic energy is less than the initial kinetic energy

POWER

- **Power** is how quickly energy converted from one form to another, i.e. the rate of doing work
- Power is measured in Watts, W

$$P = \frac{W}{t}$$

P = power, Watts, W

W = work (energy transferred), J

t = time taken, s

- Worksheet: Power

EFFICIENCY

- The efficiency of an energy transform is calculated by

$$efficiency = \frac{energy\ output}{energy\ input} \times 100\%$$

or

$$efficiency = \frac{power\ output}{power\ input} \times 100\%$$

2.4 UNIFORM CIRCULAR MOTION

CIRCULAR MOTION

- **Uniform circular motion** is when an object is moving in a circle at constant speed
- Velocity changes constantly because direction changes constantly

$$speed = \frac{d}{t} = \frac{2\pi r}{T}$$

CENTRIPETAL ACCELERATION

- The velocity of an object in circular motion is always at a tangent to the circle
- The change in velocity is always directed towards the centre of the circle
- So the acceleration is always directed towards the centre

$$a_c = \frac{v^2}{r}$$

a_c = centripetal acceleration, $m.s^{-2}$

v = linear velocity (at a tangent to the circle), $m.s^{-1}$

r = radius of the circle, m

CENTRIPETAL FORCE

- Since an object moving in circular motion is accelerating towards the centre of the circle, there must be a net force directed towards the centre of the circle

$$F_c = \frac{mv^2}{r}$$

F_c = centripetal force, N

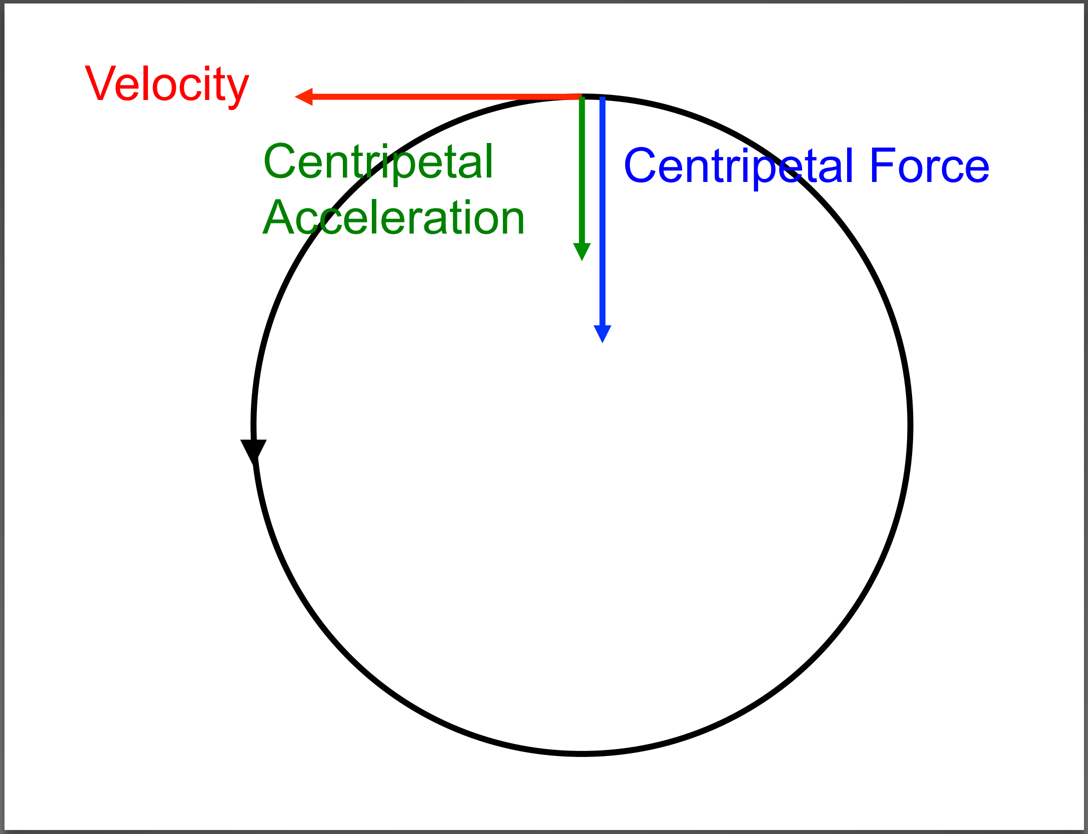
a_c = centripetal acceleration, $m.s^{-2}$

m = mass, kg

v = tangential velocity, $m.s^{-1}$

r = radius, m

VECTORS IN CIRCULAR MOTION



EXAMPLES OF CENTRIPETAL FORCES

Example of Circular Motion	Source of Centripetal Force
A mass spinning on a string	Tension in the string
A planet revolving around the Sun	Gravitational attraction between the Sun and the planet
A car driving around a corner	Friction between the tyres and the road
An aeroplane doing a vertical loop	Gravity (top of loop) and lift (bottom of loop)

6.1 GRAVITATIONAL FORCE AND FIELD

NEWTON'S LAW OF GRAVITATION

$$F_G = \frac{GMm}{r^2}$$

F_g = gravitational force, N

M = $mass_1$, kg

m = $mass_2$, kg

r = distance, m

G = gravitational constant = 6.67×10^{-11} $N.m^2kg^{-2}$

VELOCITY OF AN ORBITING SATELLITE

- For an orbiting satellite, gravity provides the centripetal force

$$F_G = F_C$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

- So velocity depends only on the height of the orbit and the mass of the central object

v = velocity of orbiting satellite, $m.s^{-1}$

r = distance between the centres of mass, m

M = mass of the central object, kg

G = gravitational constant

GRAVITATIONAL FIELD STRENGTH

- Fields have magnitude and direction
- A **gravitational field** is a region of space where a force is felt on a mass
- Gravitational field strength is the force per unit mass

$$g = \frac{F_W}{m}$$

g = gravitational field strength, $N.kg^{-1}$ (or $m.s^{-2}$)

F_w = weight force, N

m = mass of the object in the gravitational field, kg

GRAVITATIONAL FIELD OF A POINT MASS

- Since weight force is given by gravitational force

$$F_W = F_G$$

$$\frac{GMm}{r^2} = mg$$

$$g = \frac{GM}{r^2}$$

- So gravitational field strength is proportional to the mass of the central object and proportional to the inverse square of the distance from the object

g = gravitational field strength, $m.s^{-2}$

m = mass in the gravitational field, kg

r = distance from mass M, m

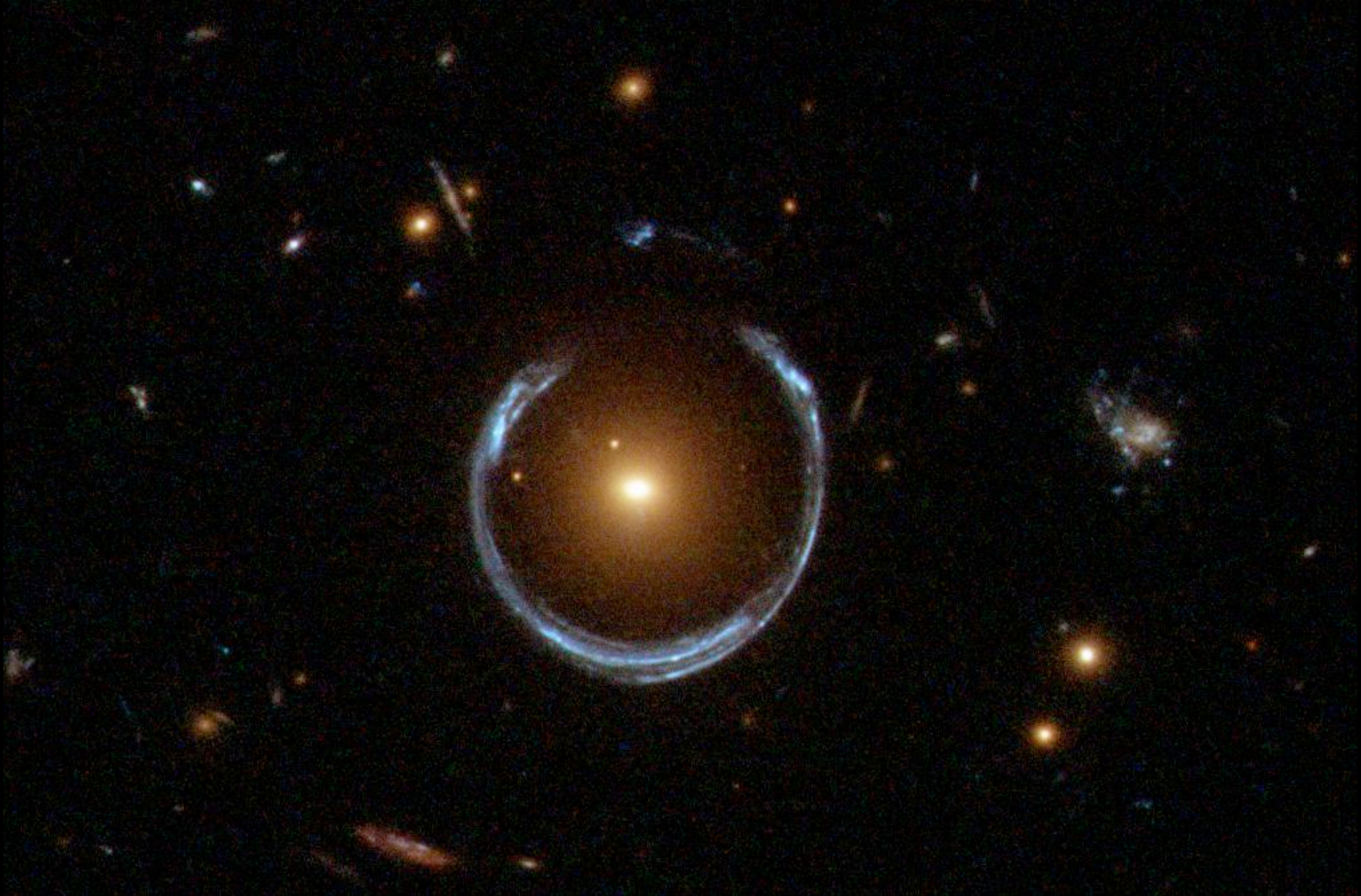
F = force, N

M = mass of the central object, kg

G = gravitational constant

GRAVITATIONAL FIELD OF MULTIPLE MASSES

- If we are considering the gravitational field that occurs as a result of more than one point mass, we use vector addition to find the resultant gravitational field
- To find the gravitational field strength at the surface of a (spherical) planet, consider the planet to be a point mass at the centre of mass of the planet, and the radius to be the radius of the planet



Gravitational micro-lensing – the gravitational field of a red galaxy has distorted the light of a more distant blue galaxy